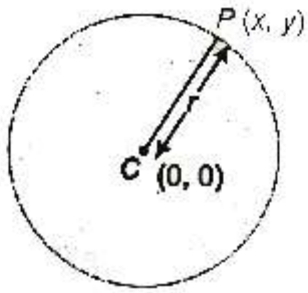
Maths Class 11 Chapter 11 Conic section part-1 Circles

Circles

Circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is constant.

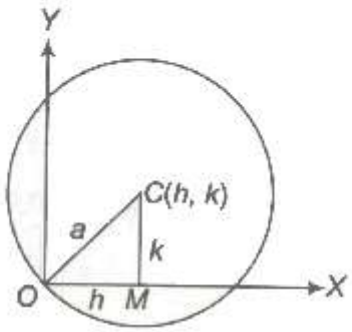


Standard Forms of a Circle

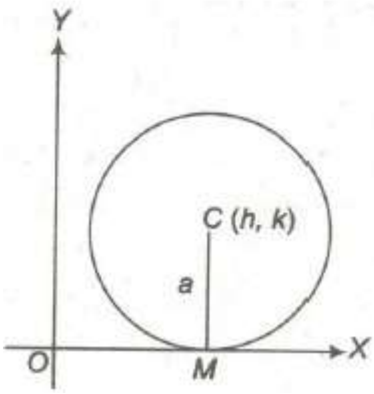
(i) Equation of circle having centre (h, k) and radius $(x — h)^2 + (y — k)^2 = a^2$.

If centre is (0, 0), then equation of circle is $x^2 + y^2 = a^2$.

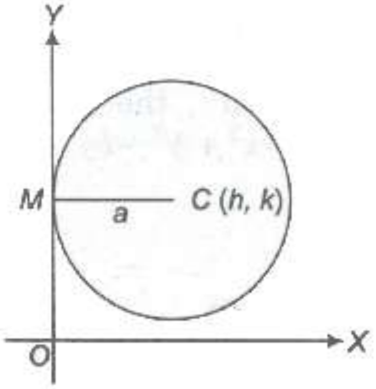
(ii) When the circle passes through the origin, then equation of the circle is $x^2 + y^2 — 2hx — 2ky = 0$.



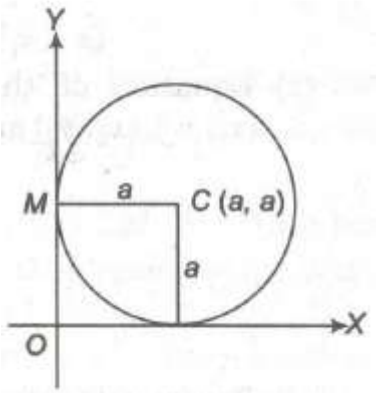
(iii) When the circle touches the X-axis, the equation is $x^2 + y^2 — 2hx — 2ay + h^2 = O$.



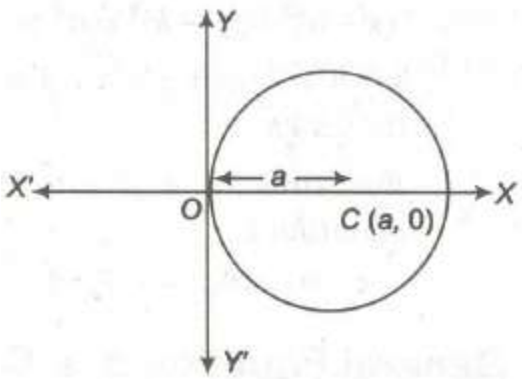
(iv) Equation of the circle, touching the Y-axis is $x^2 + y^2 — 2ax — 2ky + k^2 = 0$.



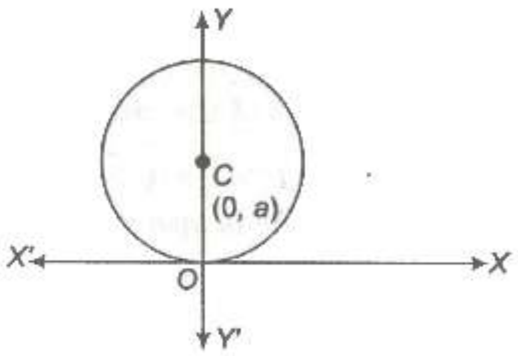
(v) Equation of the circle, touching both axes is $x^2 + y^2 — 2ax — 2ay + a^2 = O$.



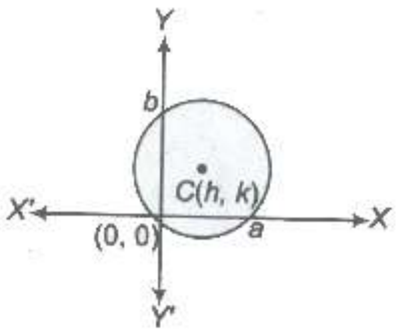
(vi) Equation of the circle passing through the origin and centre lying on the X-axis is $x^2 + y^2 — 2ax = O$.



(vii) Equation of the circle passing through the origin and centre lying on the Y-axis is $x^2 + y^2 – 2ay = 0$.



(viii) Equation of the circle through the origin and cutting intercepts a and b on the coordinate axes is $x^2 + y^2 — by = 0$.



(ix) Equation of the circle, when the coordinates of end points of a diameter are (x_1, y_1) and (x_2, y_2) is

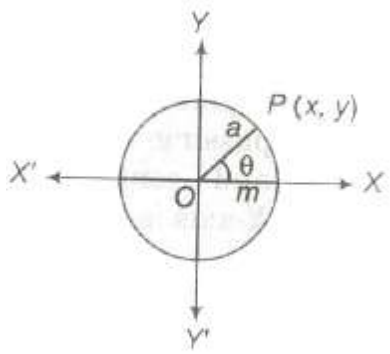
$(x — x_1)(x — x_2) + (y – y_1)(y — y_2) = 0$.

(x) Equation of the circle passes through three given points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0.$$

(xi) Parametric equation of a circle

$(x – h)^2 + (y – k)^2 = a^2$ is
$x = h + a \cos\theta$, $y = k + a \sin\theta$,
$0 \le \theta \le 2\pi$
For circle $x^2 + y^2 = a^2$, parametric equation is
$x = a \cos \theta$, $y = a \sin \theta$



General Equation of a Circle

The general equation of a circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$, where centre of the circle = (- g, – f)

Radius of the circle = $\sqrt{g^2 + f^2 - c}$

1. If $g^2 + f^2 - c > 0$, then the radius of the circle is real and hence the circle is also real.
2. If $g^2 + f^2 - c = 0$, then the radius of the circle is 0 and the circle is known as point circle.
3. If $g^2 + f^2 - c < 0$, then the radius of the circle is imaginary. Such a circle is imaginary, which is not possible to draw.

Position of a Point with Respect to a Circle

A point (x_1, y_1) lies outside on or inside a circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, according as $S_1 >, =$ or < 0
where, $S_1 = x_1^2 + y_1^2 + 2gx_1, + 2fy_1 + c$

Intercepts on the Axes

The length of the intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with X and Y-axes are

$2\sqrt{g^2 - c}$ and $2\sqrt{g^2 - c}$.

1. If $g^2 > c$, then the roots of the equation $x^2 + 2gx + c = 0$ are real and distinct, so the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets the X-axis in two real and distinct points.
2. If $g^2 = c$, then the roots of the equation $x^2 + 2gx + c = 0$ are real and equal, so the circle touches X-axis, then intercept on X-axis is O.
3. If $g^2 < c$, then the roots of the equation $x^2 + 2gx + c = 0$ are imaginary, so the given circle does not meet X-axis in real point. Similarly, the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the Y-axis in real and distinct points touches or does not meet in real point according to $f^2 >, =$ or $< c$

Equation of Tangent

A line which touch only one point of a circle.

1. Point Form

1. The equation of the tangent at the point $P(x_1, y_1)$ to a circle $x^2 + y^2$ $2gx + 2fy + c= 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
2. The equation of the tangent at the point $P(x_1, y_1)$ to a circle $x^2 + y^2$ is $xx_1 + yy_1 = r^2$

2. Slope Form

(i) The equation of the tangent of slope m to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

are $y + f = m(x + g) \pm \sqrt{(g^2 + f^2 — c)(1 + m^2)}$

(ii) The equation of the tangents of slope m to the circle $(x – a)^2 (y – b)^2 = r^2$ are $y – b = m(x – a) \pm r\sqrt{(1 + m^2)}$ and the coordinates of the points of contact are

$$\left(a \pm \frac{mr}{\sqrt{1+m^2}}, b \mp \frac{r}{\sqrt{1+m^2}}\right).$$

(iii) The equation of tangents of slope m to the circle $x^2 + y^2 = r^2$ are $y = mx \pm r\sqrt{(1 + m^2)}$ and the coordinates of the point of contact are

$$\left(\pm \frac{rm}{\sqrt{1+m^2}}, \mp \frac{r}{\sqrt{1+m^2}}\right).$$

3. Parametric Form

The equation of the tangent to the circle $(x – a)^2 + (y – b)^2 = r^2$ at the point $(a + r \cos \theta, b + r \sin\theta)$ is $(x – a) \cos \theta + (y – b) \sin \theta = r$.

Equation of Normal

A line which is perpendicular to the tangent.

1. Point Form

1. (i) The equation of normal at the point (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
   $y – y_1 = [(y_1 + f)(x – x_1)]/(x_1 + g)$
   $(y_1 + f)x – (x_1 + g)y + (gy_1 – fx_1) = 0$
2. (ii) The equation of normal at the point (x_1, y_1) to the circle
   $x^2 + y^2 = r^2$ is $x/x_1 = y/y_1$

2. Parametric Form

The equation of normal to the circle $x^2 + y^2 = r^2$ at the point ($r \cos \theta$, $r \sin \theta$) is

$(x/r \cos \theta) = (y/r \sin \theta)$

or $y = x \tan \theta$.

Important Points to be Remembered

(i) The line $y = mx + c$ meets the circle in unique real point or touch the circle

$x_2 + y_2 + r_2$, if $r = |c/\sqrt{1} + m_2$

and the point of contacts are $\left(\frac{\pm mr}{\sqrt{1+m^2}}, \frac{\mp r}{\sqrt{1+m^2}}\right)$.

(ii) The line $lx + my + n = 0$ touches the circle $x_2 + y_2 = r_2$, if $r_2(l_2 + m_2) = n_2$.

(iii) Tangent at the point P (θ) to the circle $x_2 + y_2 = r_2$ is $x \cos \theta + y \sin \theta = r$.

(iv) The point of intersection of the tangent at the points $P(\theta_1)$ and $Q(\theta_2)$ on the circle $x^2 + y^2 = r^2$

$$x = \frac{r\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \text{ and } y = \frac{r\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$$

(v) Normal at any point on the circle is a straight line which is perpendicular to the tangent to the curve at the point and it passes through the centre of circle.

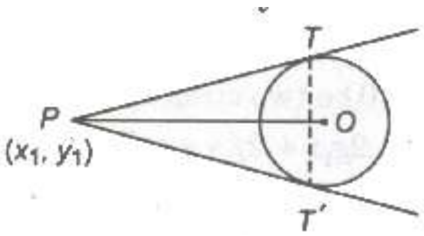
(vi) Power of a point (x_1, y_1) with respect to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.

(vii) If P is a point and C is the centre of a circle of radius r, then the maximum and minimum distances of P from the circle are CP + r and CP — r , respectively.

(viii) If a line is perpendicular to the radius of a circle at its end points on the circle, then the line is a tangent to the circle and vice-versa.

Pair of Tangents

(i) The combined equation of the pair of tangents drawn from a point $P(x_1, y_1)$ to the circle $x^2 + y^2 = r^2$ is



$(x^2 + y^2 – r^2)(x_1^2+ y_1^2 – r_1^2) = (xx_1 + yy_1 – r^2)^2$
or $SS_1 = T^2$
where, $S = x^2 + y^2 – r^2$, $S_1 = x_1^2+ y_1^2 – r_1^2$
and $T = xx_1 + yy_1 – r^2$

(ii) The length of the tangents from the point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is equal to

$$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$$

(iii) Chord of contact TT' of two tangents, drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 = r^2$ or $T = 0$.

Similarly, for the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is

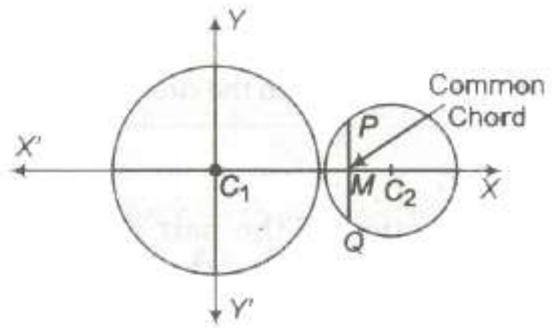
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(iv) Equation of Chord Bisected at a Given Point The equation of chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ bisected at the point (x_1, y_1) is give by $T = S_1$.
i.e., $xx_1 + yy_1 + g (x + x_1) + f (y + y_1) + c$
$= x_1^2 + y_1^2 + 2gx_1 + fy_1 + c$

(v) Director Circle The locus of the point of intersection of two perpendicular tangents to a given circle is called a director circle. For circle $x^2 + y^2 = r^2$, the equation of director circle is $x^2 + y^2 = 2r^2$.

Common Chord

The chord joining the points of intersection of two given circles is called common chord.



(i) If $S_1 = 0$ and $S_1 = 0$ be two circles, such that

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$
and $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$
then their common chord is given by $S_1 — S_2 = 0$

(ii) If C_1, C_2 denote the centre of the given circles, then their common chord

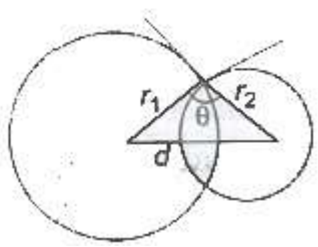
$PQ = 2\ PM = 2\sqrt{(C_1P)^2 – C_1M)^2}$

(iii) If r_1, and r_2 be the radii of ‘two circles, then length of common chord is
$\frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$.

Angle of Intersection of Two Circles

The angle of intersection of two circles is defined as the angle between the tangents to the two circles at their point of intersection is given by

$\cos \theta = (r_1^2 + r_2^2 – d^2)/(2r_1r_2)$



Orthogonal Circles

Two circles are said to be intersect orthogonally, if their angle of intersection is a right angle.

If two circles

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + C_1 = 0$ and

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + C_2 = 0$ are orthogonal, then $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Family of Circles

(i) The equation of a family of circles passing through the intersection of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and line

$L = lx + my + n = 0$ is $S + \lambda L = 0$

where, X, is any real number.

(ii) The equation of the family of circles passing through the point $A(x_1, y_1)$ and $B(x_1, y_1)$ is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(iii) The equation of the family of -circles touching the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ at point $P(x_1, Y_1)$ is

$xx^2 + y^2 + 2gx + 2fy + c + \lambda, [xx_1 + yy_1 + g(x + x_1) + f(Y+ Y_1) + c] = 0$ or $S + \lambda L = 0$, where $L = 0$ is the equation of the tangent to

$S = 0$ at (x_1, y_1) and $X \in R$

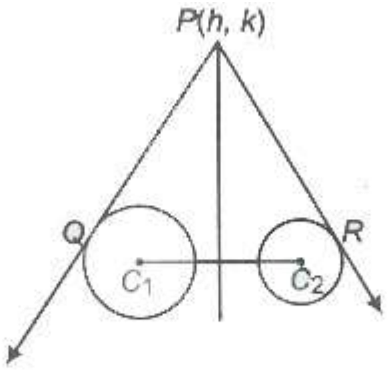
(iv) Any circle passing through the point of intersection of two circles S_1 and S_2 is $S_1 + \lambda(S_1 — S_2) = 0$.

Radical Axis

The radical axis of two circles is the locus of a point which moves in such a way that the length of the tangents drawn from it to the two circles are equal.

A system of circles in which every pair has the same radical axis is called a coaxial system of circles.

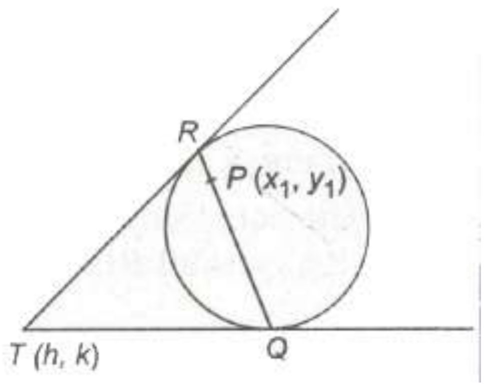
The radical axis of two circles $S_1 = 0$ and $S_2 = 0$ is given by $S_1 — S_2 = 0$.



1. The radical axis of two circles is always perpendicular to the line joining the centres of the circles.
2. The radical axis of three vertices, whose centres are non-collinear taken in pairs of concurrent.
3. The centre of the circle cutting two given circles orthogonally, lies on their radical axis.
4. Radical Centre The point of intersection of radical axis of three circles whose centre are non-collinear, taken in pairs, is called their radical centre.

Pole and Polar

If through a point P (x_1, y_1) (within or outside a circle) there be drawn any straight line to meet the given circle at Q and R, the locus of the point of intersection of tangents at Q and R is called the polar of P and po.:.at P is called the pole of polar.



1. Equation of polar to the circle $x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$.
2. Equation of polar to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
3. Conjugate Points Two points A and B are conjugate points with respect to a given circle, if each lies on the polar of the other with respect to the circle.
4. Conjugate Lines If two lines be such that the pole of one lies on the other, then they are called conjugate lines with respect to the given circle.

Coaxial System of Circles

A system of circle is said to be coaxial system of circles, if every pair of the circles in the system has same radical axis.

1. The equation of a system of coaxial circles, when the equation of the radical axis $P \equiv lx + my + n = 0$ and one of the circle of the system $S = x^2 + y^2 + 2gx + 2fy + c = 0$, is $S + \lambda P = 0$.
2. Since, the lines joining the centres of two circles is perpendicular to their radical axis. Therefore, the centres of all circles of a coaxial system lie on a straight line, which is perpendicular to the common radical axis.

Limiting Points

Limiting points of a system of coaxial circles are the centres of the point circles belonging to the family.

Let equation of circle be $x^2 + y^2 + 2gx + c = 0$

∴ Radius of circle = $\sqrt{g^2 - c}$

For limiting point, $r = 0$

∴ $\sqrt{g^2 - c} = 0$ ⇒$g = \pm \sqrt{c}$

Thus, limiting points of the given coaxial system as $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$.

Important Points to be Remembered

(i) Circle touching a line L=O at a point (x_1, y_1) on it is

$(x - x_1)^2 + (y - y_1)^2 + XL = 0$.

(ii) Circumcircle of a A with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is

$$\frac{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)}{(x_3-x_1)(x_3-x_2)+(y_3-y_1)(y_3-y_2)} \text{ or } \frac{\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}}$$

(iii) A line intersect a given circle at two distinct real points, if the length of the perpendicular from the centre is less than the radius of the circle.

(iv) Length of the intercept cut off from the line $y = mx + c$ by the circle $x^2 + y^2 = a^2$ is

$$2\sqrt{\frac{a^2(1+m^2)-c^2}{1+m^2}}$$

(v) In general, two tangents can be drawn to a circle from a given point in its plane. If m_1 and m_2 are slope of the tangents drawn from the point $P(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$, then

$$m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - a^2} \quad \text{and} \quad m_1 \times m_2 = \frac{y_1^2 - a^2}{x_1^2 - a^2}$$

(vi) Pole of $lx + my + n = 0$ with respect to $x^2 + y^2 = a^2$ is $\left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)$.

(vii) Let $S_1 = 0$, $S_2 = 0$ be two circles with radii r_1 , r_2, then $S_1/r_1 \pm S_2/r_2 = 0$ will meet at right angle.

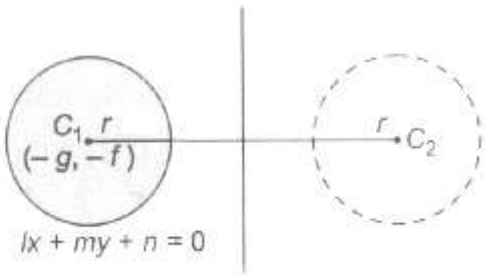
(viii) The angle between the two tangents from (x_1, y_1) to the circle $x^2 + y^2 = a^2$ is $2 \tan^{-1} (a/\sqrt{S_1})$.

(ix) The pair of tangents from (0, 0) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are at right angle, if $g^2 + f^2 = 2c$.

(x) If (x_1, y_1) is one end of a diameter of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then the other end will be $(-2g - x_1, -2f - y_1)$.

Image of the Circle by the Line Minor

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$



and line minor $lx + my + n = 0$.

Then, the image of the circle is

$(x - X_1)^2 + (y - y_1)^2 = r^2$

where, $r = \sqrt{g^2 + f^2 - c}$

Diameter of a Circle

The locus of the middle points of a system of parallel chords of a circle is called a diameter of the circle.

(i) The equation of the diameter bisecting parallel chords $y = mx + c$ of the circle $x^2 + y^2 = a^2$ is $x + my = 0$.

(ii) The diameter corresponding to a system of parallel chords of a circle always passes through the centre of the circle and is perpendicular to the parallel chords.

Common Tangents of Two Circles

Let the centres and radii of two circles are C_1, C_2 and r_1, r_2, respectively.

1. (i) When one circle contains another circle, no common tangent is possible.
   Condition, $C_1C_2 < r_1 - r_2$
2. (ii) When two circles touch internally, one common tangent is possible.
   Condition , $C_1C_2 = r_1 - r_2$
3. (iii) When two circles intersect, two common tangents are possible.
   Condition, $|r_1 - r_2| < C_1C_2 < |r_1 + r_2|$
4. (iv) When two circles touch externally, three common tangents are possible.
   Condition, $C_1C_2 = r_1 + r_2$
5. (v) When two circles are separately, four common tangents are possible.
   Condition, $C_1C_2 > r_1 + r_2$

Important Points to be Remembered

Let AS is a chord of contact of tangents from C to the circle $x^2 + y^2 = r^2$. M is the mid-point of AB.

Class 11 Maths Chapter 11 Conic Section Part -2 Ellipse

Ellipse is the locus of a point in a plane which moves in such a way that the ratio of the distance from a fixed point (focus) in the same plane to its distance from a fixed straight line (directrix) is always constant, which is always less than unity.

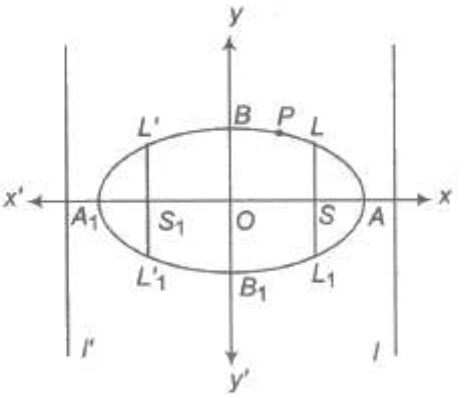
Major and Minor Axes

The line segment through the foci of the ellipse with its end points on the ellipse, is called its major axis.

The line segment through the centre and perpendicular to the major axis with its end points on the ellipse, is called its minor axis.

Horizontal Ellipse i.e., $x^2 / a^2 + y^2 / b^2 = 1, 0 < b < a$

If the coefficient of x^2 has the larger denominator, then its major axis lies along the x-axis, then it is said to be horizontal ellipse.



(i) Vertices $A(a, 0)$, $A_1 (- a, 0)$

(ii) Centre (0, 0)

(iii) Major axis, $AA_1 = 2a$; Minor axis, $BB_1 = 2b$

(iv) Foci are $S(ae, 0)$ and $S_1(-ae, 0)$

(v) Directrices are $l : x = a / e$, $l' ; x = – a / e$

(vi) Latusrectum, $LL_1 = L' L_1' = 2b^2 / a$

(vii) Eccentricity, $e = \sqrt{1 – b^2 / a^2} < 1$

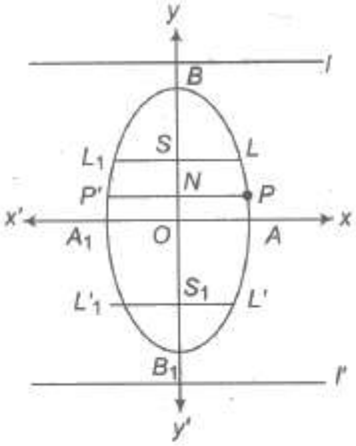
(viii) Focal distances are SP and S_1P i.e., $a – ex$ and $a + ex$. Also, $SP + S_1P = 2a$ = major axis.

(ix) Distance between foci = 2ae

(x) Distance between directrices = 2a / e

Vertical Ellipse i.e., $x^2 / a^2 + y^2 / b^2 = 1, 0 < a < b$

If the coefficient of x^2 has the smaller denominator, then its major axis lies along the y-axis, then it is said to be vertical ellipse.



(i) Vertices B(O, b), B_l(0,- b)

(ii) Centre O(0,0)

(iii) Major axis BB_l = 2b; Minor axis AA_l = 2a

(iv) Foci are S(0, ae) and S_l(0, – ae)

(v) Directrices are l : y = b / e ; l' : y = – b / e

(vi) Latusrectum LL_l = $L'L_l'$ = $2a^2 / b$

(vii) Eccentricity $e = \sqrt{1 – a^2 / b^2} < 1$

(viii) Focal distances are SP and S_lP.

i.e., b – ex and b + ex axis.

Also, SP + S_lP = 2b = major axis.

(ix) Distance between foci = 2be

(x) Distance between directrices = 2b / e

Ordinate and Double Ordinate

Let P be any point on the ellipse and PN be perpendicular to the major axis AA', such that PN produced meets the ellipse at P'. Then, PN is called the ordinate of P and PNP' is the double ordinate of P .

Special Form of Ellipse

If centre of the ellipse is (h, k) and the direction of the axes are parallel to the coordinate axes, then its equation is $(x - h)^2 / a^2 + (y - k)^2 / b^2 = 1$

Position of a Point with Respect to an Ellipse

The point (x_1, y_1) lies outside, on or inside the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ according as $x^2_1 / a^2 + y^2_1 / b^2 - 1 > 0, = \text{ or } < 0$.

Auxiliary Circle

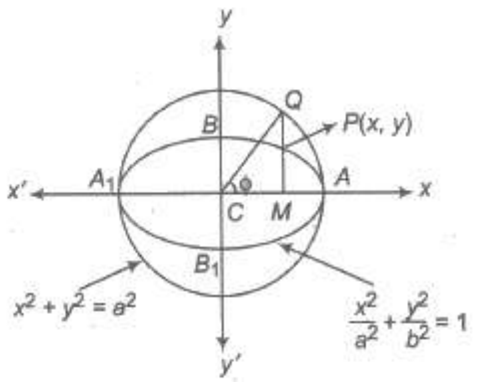
the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, becomes the ellipse $x^2 + y^2 = a^2$, if b = a. This is called auxiliary circle of the ellipse. i. e. , the circle described on the major axis of an ellipse as diameter is called auxiliary circle.

Director Circle

The locus of the point of intersection of perpendicular tangents to an ellipse is a director circle. If equation of an ellipse is $x^2 / a^2 + y^2 / b^2 = 1$, then equation of director circle is $x^2 + y^2 = a^2 + b^2$.

Eccentric Angle of a Point

Let P be any point on the ellipse $x^2 / a^2 + y^2 / b^2 = 1$. Draw PM perpendicular a b from P on the major axis of the ellipse and produce MP to the auxiliary circle in Q. Join CQ. The $\angle ACQ = \varphi$ is called the eccentric angle of the point P on the ellipse.



Parametric Equation

The equation $x = a \cos \varphi$, $y = b \sin \varphi$, taken together are called the parametric equations of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, where φ is any parameter.

Equation of Chord

Let P(a cos θ, b sin θ) and Q(a cos φ, b sin φ) be any two points of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$.

(i) The equation of the chord joining these points will be

$(y – b \sin θ) = b \sin φ – b \sin θ / a \cos φ – a \sin θ (x – a \cos θ)$

or $x / a \cos (θ + φ / 2) + y / b \sin (θ + φ / 2) = \cos (θ – φ / 2)$

(ii) The equation of the chord of contact of tangents drawn from an point (x_1, y_1) to the ellipse

$x^2 / a^2 + y^2 / b^2 = 1$ is $xx_1 / a^2 + yy_1 / b^2 = 1$.

(iii) The equation of the chord of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ bisected at the point (x_1, y_1) is given by

$xx_1 / a^2 + yy_1 / b^2 – 1 = x^2_1 / a^2 + y^2_1 / b^2 – 1$

or $T = S_1$

Equation of Tangent

(i) **Point Form** The equation of the tangent to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ at the point (x_1, y_1) is $xx_1 / a^2 + yy_1 / b^2 = 1$.

(ii) **Parametric Form** The equation of the tangent to the ellipse at the point (a cos θ, b sin θ) is $x / a \cos θ + y / b \sin θ = 1$.

(iii) **Slope Form** The equation of the tangent of slope m to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ are $y = mx ± \sqrt{a^2m^2 + b^2}$ and the coordinates of the point of contact are

$$\left(\pm \frac{a^2 m}{\sqrt{a^2m^2 + b^2}}, \mp \frac{b^2}{\sqrt{a^2m^2 + b^2}}\right).$$

(iv) **Point of Intersection of Two Tangents** The equation of the tangents to the ellipse at points $P(a \cos θ_1, b \sin θ_1)$ and $Q (a \cos θ_2, b \sin θ_2)$ are

$x / a \cos θ_1 + y / b \sin θ_1 = 1$ and $x / a \cos θ_2 + y / b \sin θ_2 = 1$

and these two intersect at the point

$$\left(\frac{a \cos\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}, \frac{b \sin\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)} \right)$$

Equation of Normal

(i) **Point Form** The equation of the normal at (x_1, y_1) to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is

$a^2x / x_1 + b^2y / y_1 = a^2 – b^2$

(ii) **Parametric Form** The equation of the normal to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ at $(a \cos \theta, b \sin \theta)$ is

$ax \sec \theta – by \operatorname{cosec} \theta = a^2 – b^2$

(iii) **Slope Form** The equation of the normal of slope m to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ are given by $y = mx – m (a^2 – b^2) / \sqrt{a^2 + b^2m^2}$

and the coordinates of the point of contact are

$$\left(\pm \frac{a^2}{\sqrt{a^2 + b^2m^2}}, \pm \frac{b^2m}{\sqrt{a^2 + b^2m^2}} \right)$$

(iv) P**oint of Intersection of Two Normals** Point of intersection of the normal at points $(a \cos \theta_1, b \sin \theta_1)$ and $(a \cos \theta_2, b \sin \theta_2)$ are given by

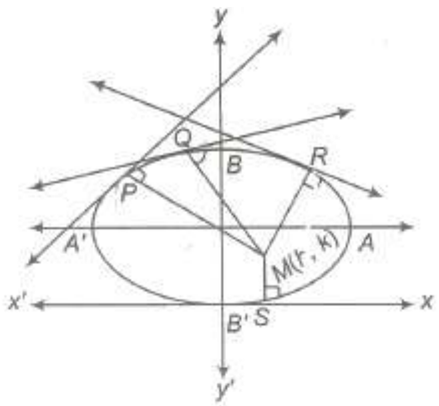
$$\left(\frac{a^2 - b^2}{a} \cos\theta_1 \cos\theta_2 \frac{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}, \frac{-(a^2 - b^2)}{b} \sin\theta_1 \sin\theta_2 \frac{\sin\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)} \right)$$

(v) If the line $y = mx + c$ is a normal to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, then

$c^2 = m^2(a^2 – b^2)^2 / a^2 + b^2m^2$

Conormal Points

The points on the ellipse, the normals at which the ellipse passes through a given point are called conormal points.



Here, P, Q, R and S are the conormal points.

(i) The sum of the eccentric angles of the conormal points on the ellipse, $x^2 / a^2 + y^2 / b^2 = 1$ is an odd multiple of π.

(ii) If θ_1, θ_2, θ_3 and θ_4 are eccentric angles of four points on the ellipse, the normals at which are concurrent, then

(a) $\Sigma \cos (\theta_1 + \theta_2) = 0$

(b) $\Sigma \sin (\theta_1 + \theta_2) = 0$

(iii) If θ_1, θ_2 and θ_3 are the eccentric angles of three points on the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, such that

$\sin (\theta_1 + \theta_2) + \sin (\theta_2 + \theta_3) + \sin (\theta_3 + \theta_1) = 0$,

then the normal at these points are concurrent.

(iv) If the normal at four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ on the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, are concurrent, then

$(x_1 + x_2 + x_3 + x_4) (1 / x_1 + 1 / x_2 + 1 / x_3 + 1 / x_4) = 4$

Diameter and Conjugate Diameter

The locus of the mid-point of a system of parallel chords of an ellipse is called a diameter, whose equation of diameter is

$y = -(b^2 / a^2 m) x$

Two diameters of an ellipse are said to be conjugate diameters, if each bisects the chords parallel to the other.

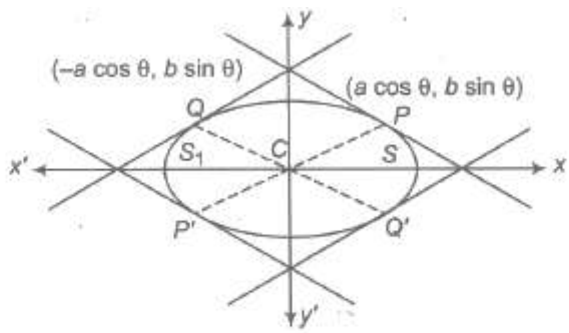
Properties of Conjugate Diameters

(i) The eccentric angles of the ends of a pair of conjugate diameters of an ellipse differ by a right angle.

(ii) The sum of the squares of any two conjugate semi-diameters of an ellipse is constant and equal to the sum of the squares of the semi-axis of the ellipse i. e., $CP^2 + CD^2 = a^2 + b^2$.

(iii) If CP, CQ are two conjugate semi-diameters of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$ and S, S_1 be two foci of an ellipse, then

$SP * S_1P = CQ^2$



(iv) The tangent at the ends of a pair of conjugate diameters of an ellipse form a parallelogram.

(v) The area of the parallelogram formed by the tangents at the ends of conjugate diameters of an ellipse is constant and is equal to the product of the axis.

Important Points

1. The point $P(x_1\ y_1)$ lies outside, on or inside the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ according as $x^2_1 / a^2 + y^2_1 / b^2 – 1 > 0$, or < 0.

2. The line $y = mx + c$ touches the ellipse

$x^2 / a^2 + y^2 / b^2 = 1$, if $c^2 = a^2m^2 + b^2$

3. The combined equation of the pair of tangents drawn from a point $(x_1\ y_1)$ to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is

$(x^2 / a^2 + y^2 / b^2 – 1)\ (x^2_1 / a^2 + y^2_1 / b^2 – 1) = (xx_1 / a^2 + yy_1 / b^2 – 1)^2$

i.e, $SS_1 = T^2$

4. The tangent and normal at any point of an ellipse bisect the external and internal angles between the focal radii to the point.

5. If SM and S’ M’ are perpendiculars from the foci upon the tangent at any point of the ellipse, then SM x S’ M’ = b^2 and M, M’ lie on the auxiliary circle.

6. If the tangent at any point P on the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ meets the major axis in T and minor axis in T', then CN * CT = a^2 ,CN' * Ct' = p^2, where N and N' are the foot of the perpendiculars from P on the respective axis.

7. The common chords of an ellipse and a circle are equally inclined to the axes of the ellipse.

8. The four normals can be drawn from a point on an ellipse.

9. Polar of the point $(x_1\ y_1)$ with respect to the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is $xx_1 / a^2 + yy_1 / b^2 = 1$.

Here, point $(x_1\ y_1)$ is the pole of $xx_1 / a^2 + yy_1 / b^2 = 1$.

10. The pole of the line $lx + my + n = 0$ with respect to ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is

$p(-a^2l / n, -b^2m / n)$.

11. Two tangents can be drawn from a point P to an ellipse. These tangents are real and distinct or coincident or imaginary according as the given point lies outside, on or inside the ellipse.

12. Tangents at the extremities of latusrectum of an ellipse intersect on the corresponding direction.

13. Locus of mid-point of focal chords of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is

$x^2 / a^2 + y^2 / b^2 = ex / a^2$.

14. Point of intersection of the tangents at two points on the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, whose eccentric angles differ by a right angles lies on the ellipse $x^2 / a^2 + y^2 / b^2 = 2$.

15. Locus of mid – point of normal chords of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$ is

$(x^2 / a^2 + y^2 / b^2)^2 (a^6 / x^2 + b^6 / y^2) = (a^2 – b^2)^2$.

16. Eccentric angles of the extremities of latusrectum of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$ are

$\tan^{-1} (\pm b / ae)$.

17. The straight lines $y = m_1x$ and $y =m_2x$ are conjugate diameters of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$, if $m_1m_2 = – b^2 / a^2$.

18. The normal at point P on an ellipse with foci S, S_1 is the internal bisector of $\angle SPS_1$.

Class 11 Maths Chapter 11 Conic Section Part -2 Hyperbola

A hyperbola is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point in the same plane to its distance from a fixed line is always constant, which is always greater than unity.

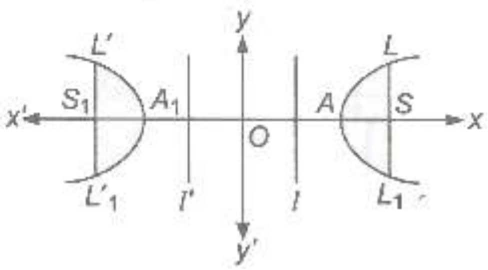
The fixed point is called the focus and the fixed line is directrix and the ratio is the eccentricity.

Transverse and Conjugate Axes

The line through the foci of the hyperbola is called its transverse axis.

The line through the centre and perpendicular to the transverse axis of the hyperbola is called its conjugate axis.

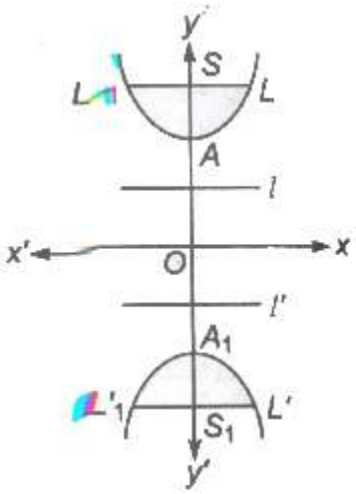
Hyperbola of the Form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



1. Centre O(0, 0)
2. Foci are S(ae,0),S_1(-ae, 0)
3. Vertices A(a, 0), A_1(-a, 0)
4. Directrices / : x = a/e, l' : x = -a/e
5. Length of latusrectum $LL_1 = L'L'_1 = 2b^2/a$
6. Length of transverse axis 2a.
7. Length of conjugate axis 2b.
8. Eccentricity $e = \sqrt{1 + \left(\frac{a}{b}\right)^2}$ or $b^2 = a^2(e^2 - 1)$
9. Distance between foci =2ae
10. Distance between directrices = 2a/e

Conjugate Hyperbola

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



1. (i) Centre O(0, 0)
2. (ii) Foci are S (0, be), S_1(0, — be)
3. (iii) Vertices A(0, b) , A_1(0, — b)
4. (iv) Directrices
   l:y = b/e, l' : y = —b/e
5. (v) Length of latusrectum
   LL_1 = L' L'_1 = $2a_2$/b
6. (vi) Length of transverse axis 2b.
7. (vii) Length of conjugate axis 2a.
8. (viii) Eccentricity
   $$e = \sqrt{1 + \left(\frac{a}{b}\right)^2}$$
9. (ix) Distance between foci = 2be
10. (x) Distance between directrices = 2b/e

Focal Distance of a Point

The distance of a point on the hyperbola from the focus is called it focal distance. The difference of the focal distance of any point on a, hyperbola is constant and is equal to the length of transverse axis the hyperbola i.e.,

S_1P — SP = 2a

where, S and S_1 are the foci and P is any point or P the hyperbola.

Equation of Hyperbola in Different Form

1 If the centre of the hyperbola is (h, k) and the directions of the axes are parallel to the coordinate axes, then the equation of the hyperbola, whose transverse and conjugate axes are 2a and 2b is

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1.$$

2. If a point P(x, y) moves in the plane of two perpendicular straight lines $a_1x + b_1y + c_1 = 0$ and $b_1x – a_1y + c_2 = 0$ in such a way that

$$\frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2}-\frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{a_1^2+b_1^2}}\right)^2}{b^2}=1$$

Then, the locus of P is hyperbola whose transverse axis lies along $b_1x – a_1y + c_2 = 0$ and conjugate axis along the line $a_1x + b_1y + c_1 = 0$. The length of transverse and conjugate axes are 2a and 2b, respectively.

Parametric Equations

(i) Parametric equations of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$x = a \sec \theta$, $y = b \tan \theta$
or $x = a \cosh \theta$, $y = b \sinh\theta$

(ii) The equations $x=a\left(\frac{e^{\theta}+e^{-\theta}}{2}\right),\ y=b\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)$ are also the parametric equations of the hyperbola.

Equation of Chord

(i) Equations of chord joining two points P(a sec θ_1, b tan θ_1,) and Q(a sec θ_2, b tan θ_2) on the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$y-b\tan\theta=\frac{b\tan\theta_2-b\tan\theta_1}{a\sec\theta_2-a\sec\theta_1}\cdot(x-a\sec\theta_1)$$

or $$\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)$$

(ii) Equations of chord of contact of tangents drawn from a point (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1.$

(iii) The equation of the chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ bisected at point (x_1, y_1) is given by

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 \quad \text{or} \quad T = S_1$$

Equation of Tangent Hyperbola

(i) Point Form The equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

(ii) Parametric Form The equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a \sec\theta, b\tan\theta)$ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$.

(iii) Slope Form The equation of the tangents of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are given by $y = mx \pm \sqrt{a^2m^2 - b^2}$.

The coordinates of the point of contact are

$$\left(\pm \frac{a^2m}{\sqrt{a^2m^2 - b^2}}, \pm \frac{b^2}{\sqrt{a^2m^2 - b^2}}\right).$$

(iv) The tangent at the points P(a sec θ_1 , b tan θ_1) and Q (a sec θ_2, b tan θ_2) intersect at the point

$$\left[\frac{a\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}, \frac{b\sin\left(\frac{\theta_1 - \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}\right]$$

(v) Two tangents drawn from P are real and distinct, coincident or imaginary according as the roots of the equation $m^2(h^2 - a^2) - 2khm + k^2 + b^2 = 0$. are real and distinct, coincident or imaginary.

(vi) The line y = mx + c touches the hyperbola, if $c^2 = a^2m^2 - b^2$ the point of contacts $\left(\pm \frac{a^2m}{c}, \pm \frac{b^2}{c}\right)$, where $c = \sqrt{a^2m^2 - b^2}$.

Normal Equation of Hyperbola

(i) Point Form The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$.

(ii) Parametric Form The equation of the normal at (a sec θ, b tan θ) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
is $ax \cos\theta + by \cot\theta = a^2 + b^2$.

(iii) Slope Form The equations of the normal of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are given by

$$y = mx \mp \frac{m(a^2 + b^2)}{\sqrt{a^2 - b^2m^2}}$$

The coordinates of the point of contact are

$$\left(\pm \frac{a^2}{\sqrt{a^2 - b^2m^2}}, \mp \frac{b^2m}{\sqrt{a^2 - b^2m^2}}\right).$$

(iv) The line y = mx + c will be normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if,

$$c^2 = \frac{m^2(a^2 + b^2)^2}{a^2 - b^2m^2}$$

(v) Maximum four normals can be drawn from a point (x_1, y_1) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

Conormal Points

Points on the hyperbola, the normals at which passes through a given point are called conormal points.

1. The sum of the eccentric angles of conormal points is an odd ion multiple of π.
2. If θ_1, θ_2, θ_3 and θ_4 are eccentric angles of four points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$, then normal at which they are concurrent, then
   (a) $\sum\cos(\theta_1 + \theta_2) = 0$
   (b) $\sum\sin(\theta_1 + \theta_2) = 0$
3. If θ_1, θ_2 and θ_3 are the eccentric angles of three points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$, such that $\sin(\theta_1 + \theta_2) + \sin(\theta_2 + \theta_3) + \sin(\theta_3 + \theta_1) = 0$. Then, the normals at these points are concurrent.

4. If the normals at four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(X_4, y_4)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are concurrent, then
$$(x_1 + x_2 + x_3 + x_4)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right) = 4$$

Conjugate Points and Conjugate Lines

1. Two points are said to be conjugate points with respect to a hyperbola, if each lies on the polar of the other.
2. Two lines are said to be conjugate lines with respect to a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if each passes through the pole of the other.

Diameter and Conjugate Diameter

1. **Diameter** The locus of the mid-points of a system of parallel chords of a hyperbola is called a diameter.>
The equation of the diameter bisecting a system of parallel chord of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
$y = \frac{b^2}{a^2 m} x$
2. **Conjugate Diameter** The diameters of a hyperbola are sal to be conjugate diameter, if each bisect the chords parallel to th other.
The diameters $y = m_1x$ and $y = m_2x$ are conjugate, if $m_1 m_2 = b^2/a^2$.
3. In a pair of conjugate diameters of a hyperbola, only one mee the hyperbola in real points.

Asymptote

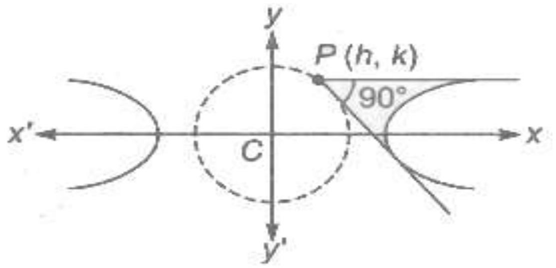
An asymptote to a curve is a straight line, at a finite distance from the origin, to which the tangent to a curve tends as the point of contact goes to infinity.

1. The equation of two asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = \pm \frac{b}{a} x$ or $\frac{x}{a} \pm \frac{y}{b} = 0$
2. The combined equation of the asymptotes to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.
3. When b = a, i.e., the asymptotes of rectangular hyperbola $x^2 - y^2 = a^2$ are $y = \pm x$ which are at right angle.
4. A hyperbola and its conjugate hyperbola have the same asymptotes.
5. The equation of the pair of asymptotes differ the hyperbola and the conjugate hyperbola by the same constant only i.e., **Hyperbola — Asymptotes = Asymptotes — Conjugate hyperbola**

6. The asymptotes pass through the centre of the hyperbola.
7. The bisectors of angle between the asymptotes are the coordinate axes.
8. The angle between the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 2 $\tan^{-1}$(b/a) or 2 $\sec^{-1}$(e).

Director Circle

The locus of the point of intersection of the tangents to the hyperbolo $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, which are perpendicular to each other, is called a director circle. The equation of director circle is $x^2 + y^2 = a^2 - b^2$.



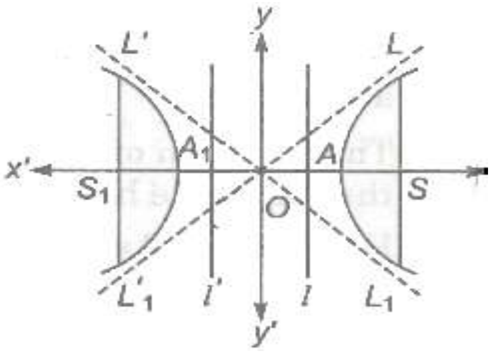
Rectangular Hyperbola

A hyperbola whose asymptotes include a right angle is said to I rectangular hyperbola or we can say that, if the lengths of transver: and conjugate axes of any hyperbola be equal, then it is said to be rectangular hyperbola.

i.e., In a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. if b = a, then it said to be rectangular hyperbola.

The eccentricity of a rectangular hyperbola is always √2.

Rectangular Hyperbola of the Form $x^2 - y^2 = a^2$

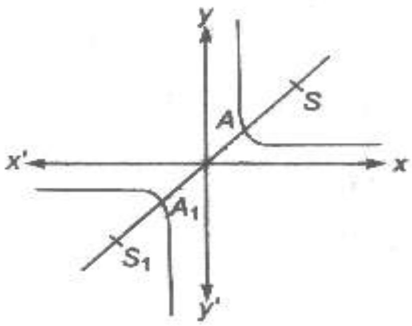


1. Asymptotes are perpendicular lines i.e., x ± y = 0
2. Eccentricity e = √2.
3. Centre (0, 0)
4. Foci (± -√2 a, 0)
5. Vertices A(a, 0) and A1 (—a, 0)

6. Directrices $x = + a/\sqrt{2}$
7. Latusrectum = 2a
8. Parametric form $x = a \sec \theta$, $y = a \tan \theta$
9. Equation of tangent, $x \sec \theta – y \tan \theta = a$

Rectangular Hyperbola of the Form $xy = c^2$

1. Asymptotes are perpendicular lines i.e., x = 0 and y = 0
2. Eccentricity $e = \sqrt{2}$
3. Centre (0, 0)
4. Foci $S(\sqrt{2}c, \sqrt{2}c)$, $S_1(-\sqrt{2}c, -\sqrt{2}c)$



5. Vertices A(c, c), A1(— c,— c)
6. Directrices $x + y = \pm\sqrt{2}c$
7. Latusrectum = $2\sqrt{2}c$
8. Parametric form x = ct, y = c/t

Tangent Equation of Rectangular Hyperbola $xy = c^2$

1. Point Form The equation of tangent at (x_1, y_1) to the rectangular hyperbola is $xy_1 + yx_1 = 2c^2$ or $(x/x_1 + y/y_1) = 2$.
2. Parametric Form The equation of tangent at (ct, c/t) to the hyperbola is(x/t + yt) = 2c.
3. Tangent at $P(ct_1, c/t_1)$ and $Q(ct_2, c/t_2)$ to the rectangular hyperbola intersect a $\left(\frac{2ct_1t_2}{t_1 + t_2}, \frac{2c}{t_1 + t_2}\right)$
4. The equation of the chord of contact of tangents drawn from a point (x_1, y_1) to the rectangular hyperbola is $xy_1 + yx_1 = 2c^2$.

Normal Equation of Rectangular Hyperbola $xy = c^2$

1. **Point Form** The equation of the normal at (x_1, y_1) to the rectangular hyperbola is $xx_1 – yy_1 = x_1^2 – y_1^2$.
2. **Parametric Form** The equation of the normal at (ct, c/t)to the rectangular hyperbola $xy = c^2$ is $xt^3 — yt — ct^4 + c = O$.
3. The equation of the normal at(ct, c/t)is a fourth degree equation t in t. So, in general four normals can be drawn from a point to the hyperbola $xy = c^2$.

Important Points to be Remembered

1. The point (x1, y1) lies outside, on or inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according as $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 <, = \text{ or } > 0$
2. The combined equation of the pairs of tangent drawn from a point $P(x_1, y_1)$ lying outside the hyperbola $S = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $SS_1 - T^2$.
3. The equation of the chord of the hyperbola $xy = c^2$ whose mid-point is (x_1, y_1) is
   $xy_1 + yx_1 = 2x_1y_1$
   or $t = S_1$
4. Equation of the chord joining t_1, t_2 on $xy = t^2$ is
   $x + yt_1t_2 = c(t_1 + t_2)$
5. Eccentricity of the rectangular hyperbola is √2 and the angle between asymptotes is 90°.
6. If a triangle is inscribed in a rectangular hyperbola, then its orthocentre lies on the hyperbola.
7. Any straight line parallel to an asymptotes of a hyperbola intersects the